International Journal of Engineering Research & Management Technology ISSN: 2348-4039

July-2017 Volume 4, Issue-4

Email: editor@ijermt.org www.ijermt.org

INVENTORY MODEL THROUGH TIME DEPENDENT DEMAND AND DETERIORATION UNDER PARTIAL BACKLOGGING

Dr.Atul Kumar Goel

Associate Professor & Head

Department of Mathematics

A.S.(P.G.) College Mawana, Meerut

ABSTRACT

Deterioration of physical goods in stock is very realistic feature of inventory control because there are many goods that either deteriorate or obsolete in the course of time. Deterioration rate of any item is either constant or time dependent. When deterioration is time dependent, time is accompanied by proportional loss in the value of the product. Realization of this factor motivated modelers to consider the deterioration factor as one of the modeling aspects. In this paper we developed a general inventory model for deteriorating items with constant deterioration rate under the consideration of time dependent demand rate and partial backlogging.

KEY WORDS: Deterioration, partial backlogging

INTRODUCTION

the recent years there is a state of interest of studying time dependent demand rate. It is observed that the demand rate of newly launched products such as electronics items, mobile phones and fashionable garments increases with time and later it becomes constant. Deterioration of items cannot be avoided in business scenarios. In most of the cases the demand for items increases with time and the items that are stored for future use always loose part of their value with passage of time. In inventory this phenomenon is known as deterioration of items. The rate of deterioration is very small in some items like hardware, glassware, toys and steel. The items such as medicine, vegetables, gasoline alcohol, radioactive chemicals and food grains deteriorate rapidly over time so the effect of deterioration of physical goods cannot be ignored in many inventory systems. The deterioration of goods is a realistic phenomenon in many inventory systems and controlling of deteriorating items becomes a measure problem in any inventory system. Due to deterioration the problem of shortages occurs in any inventory system and shortage is a fraction that is not available to satisfy the demand of the customers in a given period of time. Dye [2002] developed an inventory model with partial backlogging and stock dependent demand. Chakrabarty et al. [1998] extended the Philip's model [1974]. Skouri and Papachristors [2003] determine an optimal time of an EOQ model for deteriorating items with time dependent partial backlogging. Manjusri Basu and Sudipta Sinha [2007] extended the Yan and Cheng model [1998] for time dependent backlogging rate. Rau et al. [2004] considered an inventory model for determining an economic ordering policy of deteriorating items in a supply chain management system. Teng and Chang [2005] determined an economic production quantity in an inventory model for deteriorating items. Dave and Patel [1983] developed an inventory model together with an instantaneous replenishment policy for deteriorating items with time proportional demand and no shortage. Roy and Chaudhury [1983] considered an order level inventory model with finite rate of replenishment and allowing shortages.

Mishra [1975], Dev and Chaudhuri [1986] assumed time dependent deterioration rate in their models. In this regard an extended summary was given by Raafat[1991]. Berrotoni [1962] discussed the difficulties of fitting empirical data to mathematical distributions. Covert and Philip [1973] developed an inventory model for deteriorating items by considering two parameters weibull distribution. Mandal and Phaujdar [1989] developed a production inventory model for deteriorating items with stock dependent demand and uniform rate of production. In this direction some work also done by Padmanabhan and Vrat [1995]. Ray and Chaudhuri [1997], Mondal and Moiti [1999], Biermans and Thomas [1997], Buzacoh [1975], Chandra and Bahner [1988], Jesse et al. [1983], Mishra [1979] developed their models and show the effect of inflation in inventory models by taking a constant rate of inflation. Liao et al [2000] discuss the effect of permissible delay in payment for an inventory model of deteriorating items under inflation. Bhahmbhatt [1982] developed an EOQ model under price dependent inflation rate. Ray and Chaudhuri [1997] considered an EOQ model with shortages under the effect of inflation and time discount. Goyal [1985] developed an EOQ model under the conditions of permissible delay in payment. Chung et al [2002] and Hung [2003] considered an optimal replenishment policy for EOQ model under permissible delay in payments. Aggarwal and Jaggi [1995] extended the EOQ model with constant rate of deterioration. Hwang and Shinn [1997] determined the lot size policy for the items with exponential demand and permissible delay in payment. In the presence of trade credit policy, Chung and Hung [2005] developed an EOQ model. Vinod kumar Mishra and Lal sahab singh 2010] developed an inventory model for deteriorating items with time dependent demand and partial backlogging. Further Vinod kumar Mishra [2013] developed an inventory model involving controllable deterioration rate to extend the traditional EOQ model. Mandal [2013] developed an inventory model for random deteriorating items with timedependent demand and partial backlogging. It has been observed that the unsatisfied demand is completely back-logged and during the shortage period either all the customers wait for the arrival of next order (completely backlogged) or all the customers leave the system (completely lost). The length of waiting time for the replenishment is the main factor for determining whether the backlogging is accepted or not.

ASSUMPTIONS AND NOTATIONS

To develop an inventory model with variable demand and partial backlogging the following notations and assumptions are used:

Assumptions

Demand rate is taken as linear.

Deterioration rate is time dependent.

Shortages are allowed with partial backlogging.

Backlogging rate is an exponential decreasing function of time.

Replenishment rate is infinite.

A single item is considered over the prescribed interval.

There is no repair or replenishment of deteriorated units.

Notations

$I(t)$ the inventory level at time t.

θt variable rate of defective units out of on hand inventory at time t, $0 < \theta << 1$.

C' the inventory ordering cost per order.

C_1 are the holding cost per unit per unit time

C_2 unit purchase cost per unit

C_3 shortage cost per unit per unit time

C_4 lost sale cost per unit per unit time

t_1 is the time at which shortage starts and T is the length of replenishment cycle. $0 \le t_1 \le T$.

$f(t) = a + bt$

The variable demand rate is, a > 0, b > 0.

Here a is initial rate of demand, b is the rate with which the demand rate increases.

$\exp(-\delta t)$ Unsatisfied demand is backlogged at a rate, the backlogging parameter δ is a positive constant.

FORMULATION AND SOLUTION OF THE MODEL

The depletion of inventory during the interval (0, t_1) is due to joint effect of demand and deterioration of items and the demand is partially backlogged in the interval (t_1, T). The differential equations describing the inventory level I (t) in the interval (0, T) are given by

$$I'(t)+\theta t\ I(t)=-f(t),\ 0\le t\le t_1 \qquad \dots (1)$$

$$I'(t)=-f(t)\ e^{-\delta t},\ t_1\le t\le T \qquad \dots (2)$$

with the conditions, I (t_1) = 0 and I (0) = S … (3)

The solutions of equations (1) and (2) can be obtained as

$$I(t)=a(t_1-t)+\frac{b}{2}(t_1^2-t^2)+\frac{a\theta}{6}(t_1^3-3t_1t^2+2t^3)+\frac{b\theta}{8}(t_1^2-t^2)^2,\ 0\le t\le t_1 \qquad \dots (4)$$

and $$I(t)=\left\{a\delta^2+b\delta(\delta t+1)\right\}\frac{e^{-\delta t}}{\delta^3}-\left\{a\delta^2+b\delta(\delta t_1+1)\right\}\frac{e^{-\delta t_1}}{\delta^3},\ t_1\le t\le T \qquad \dots (5)$$

Also the initial inventory level

$$S=at_1+\frac{b}{2}t_1^2+\frac{a\theta}{2}\frac{t_1^3}{3}+\frac{b\theta t_1^4}{8} \qquad \dots (6)$$

The inventory holding cost (C_H) per cycle is given by

$$C_H=C_1\int_0^{t_1}I(t)dt=C_1\left(\frac{at_1^2}{2}+\frac{bt_1^3}{3}+\frac{a\theta t_1^4}{12}+\frac{b\theta t_1^5}{15}\right) \qquad \dots (7)$$

The deterioration cost (C_D) per cycle is given by

$$C_D=C_2\left\{I(0)-\int_0^{t_1}f(t)dt\right\}=C_2\left\{\frac{a\theta t_1^3}{6}+\frac{b\theta t_1^4}{8}\right\} \qquad \dots (8)$$

The shortage cost (C_S) per cycle due to backlog is given by

$$C_S=-C_3\int_{t_1}^{T}I(t)dt=\frac{C_3}{\delta^4}\left\{a\delta^2+b\delta(2+\delta T)\right\}e^{-\delta T}$$

$$-\frac{C_3}{\delta^4}\left[a\delta^2\left\{1-\delta(T-t_1)\right\}+b\delta\left\{(2-\delta T)(1+\delta t_1)+\delta^2t_1^2\right\}\right]e^{-\delta t_1} \qquad \dots (9)$$

and the opportunity cost (C_0) per cycle due to lost sales is given by

$$C_0=C_4\int_{t_1}^{T}(1-e^{-\delta t})(a+bt)dt$$

$$=C_4\left[a(T-t_1)+\frac{b}{2}(T^2-t_1^2)+\frac{1}{\delta^3}\left\{a\delta^2+b\delta(1+\delta T)\right\}e^{-\delta T}\right.$$

$$\left.-\frac{1}{\delta^3}\left\{a\delta^2+b\delta(1+\delta t_1)\right\}\right]e^{-\delta t_1} \qquad \dots (10)$$

Hence, the total average cost of the system is given by

$$TC=\frac{1}{T}(C'+C_H+C_D+C_S+C_O) \qquad \dots (11)$$

$$=\frac{1}{T}[C'+C_1\left(\frac{at_1^2}{2}+\frac{bt_1^3}{3}+\frac{a\theta t_1^4}{12}+\frac{b\theta t_1^5}{15}\right)+C_2\left\{\frac{a\theta t_1^3}{6}+\frac{b\theta t_1^4}{8}\right\}$$

$$+\frac{C_3}{\delta^4}\left\{a\delta^2+b\delta(2+\delta T)\right\}e^{-\delta T}$$

$$-\frac{C_3}{\delta^4}\left[a\delta^2\left\{1-\delta(T-t_1)\right\}+b\delta\left\{(2-\delta T)(1+\delta t_1)+\delta^2 t_1^2\right\}\right]e^{-\delta t_1}$$
$$+C_4[a(T-t_1)+\frac{b}{2}(T^2-t_1^2)+\frac{1}{\delta^3}\left\{a\delta^2+b\delta(1+\delta T)\right\}e^{-\delta T}$$
$$-\frac{1}{\delta^3}\left\{a\delta^2+b\delta(1+\delta t_1)+c(2+2\delta t_1+\delta^2 t_1^2)\right\}e^{-\delta t_1}]]$$

To minimize total average cost per unit time, the optimal values of t_1 and T can be obtained by solving the following equations simultaneously

$$\frac{\partial TC}{\partial t_1}=0 \qquad \dots (12)$$

and $$\frac{\partial TC}{\partial T}=0 \qquad \dots (13)$$

provided they satisfy the following conditions

$$\left.\begin{array}{l}\dfrac{\partial^2 TC}{\partial t_1^2}>0,\dfrac{\partial^2 TC}{\partial T^2}>0\\ \text{and}\quad\left(\dfrac{\partial^2 TC}{\partial t_1^2}\right)\left(\dfrac{\partial^2 TC}{\partial T^2}\right)-\left(\dfrac{\partial^2 TC}{\partial t_1\partial T}\right)^2>0\end{array}\right\} \qquad \dots (14)$$

The numerical solution of these equations can be obtained by using some suitable computational numerical method.

SENSITIVITY ANALYSIS

Table 1: Variation in system parameters

Parameter	%	-50	-25	0	25	50
A	t_1	0.89398	0.89441	0.89456	0.89463	0.89479
	T	2.7281	2.7646	2.7834	2.8067	2.8193
	TC	57.8903	58.1145	58.2408	58.8923	59.0632
B	t_1	0.89362	0.89435	0.89456	0.89489	0.89496
	T	2.7190	2.7578	2.7834	2.8124	2.8347
	TC	55.6719	56.7891	58.2408	59.6720	60.8914
δ	t_1	0.82624	0.84495	0.89456	0.91672	0.93781
	T	2.7365	2.7645	2.7834	2.7964	2.8352
	TC	54.3672	55.7823	58.2408	60.4721	62.2574
θ	t_1	0.93789	0.91681	0.89456	0.85782	0.82163
	T	2.8289	2.7923	2.7834	2.7735	2.7379
	TC	53.6705	55.2289	58.2408	61.7820	63.8203

CONCLUSION

This paper strives; an inventory model for a decaying item with linear demand. We allow the shortages with partial backlogging in this model and backlogging rate is an exponential decreasing function of time. From the analysis of model, it has been concluded that if the demand parameters are increases then the time and total cost are increases. If the deterioration parameter is increases then the time is decreases and total cost is increases. We use a numerical example to illustrate the model and sensitivity analysis. Also, the effects of changes of different parameters are studied graphically on the average cost. A natural extension of this research is to consider finite replenishment. Also we extend the deterministic demand function to stochastic demand patterns. Furthermore, we could generalize the model to allow for permissible delay in payments which are more suited to present-day market conditions. Hence, from the economical point of view, the proposed model will be useful to the business houses in the present context as it gives better inventory control system.

REFERENCES

1. Aggarwal, S.P. and Jaggi, C.K. (1995), Ordering policies of Deteriorating items under permissible delay in payments. Journal of Operational Research Society 46, 658-662.
2. Berrotoni, J.N. (1962) Practical Applications of Weibull distribution ASQC. Tech. Conf. Trans. 303-323.
3. Covert, R.P. and Philip, G.C. (1973) An EOQ model for deteriorating items with weibull distributions deterioration, AIIE Trans 5, 323- 332.
4. Buzacott, J.A. (1975) Economic order quantities with inflation. Operational Research Quarterly 26, 1188-1191.
5. Mishra, R.B. (1975) Optimum production lotsize model for a system with deteriorating inventory, Int. J. Prod. Res. 13, 161-165.
6. Biermans, H. and Thomas, J. (1977) Inventory decisions under inflationary conditions Decision Sciences 8, 151-155.
7. Bhahmbhatt, A.C. (1982) Economic order quantity under variable rate inflation and mark-up prices Productivity 23, 127-130.
8. Jesse, R.R., Mitra, A. and Cox, J.F. (1983) EOQ formula is it valid under inflationary conditions? Decision Sciences 14(3), 370-374.
9. Dave, U. and Patel, L.K. (1983) (T.So.) policy inventory model for deteriorating items with time proportional demand. J. Opl. Res. Soc. 20, 99-106.
10. Bansal K.K., Ahalawat N. [2012] Inventory System with Stock Dependent Demand and Partial Backlogging: A Critical Analysis. *Kushagra International Management Review 2[2] 94*
11. Bansal K.K., Ahlawat N. [2012] Integrated Inventory Models for Decaying Items with Exponential Demand under Inflation. *International Journal of Soft Computing and Engineering (IJSCE) 2[3] 578-587*
12. Goyal, S.K. (1985) Economic order quantity under conditions of permissible delay in payments. Journal of Operational Research Society 36, 335-338.
13. Deb, M. and Chaudhuri K.S. (1986) An EOQ model for items with finite of production and variable rate of deterioration. Opsearch 23, 175-181.
14. Kumar A., Bansal K.K.[2014] A Deterministic Inventory Model for a Deteriorating Item Is Explored In an Inflationary Environment for an Infinite Planning Horizon. *International Journal of Education and Science Research Review1 [4] 79-86*
15. Sharma M.K.,Bansal K.K.[2017] Inventory Model for Non-Instantaneous Decaying Items with Learning Effect under Partial Backlogging and Inflation. *Global Journal of Pure and Applied Mathematics.vol.13 [6]* pp. 1999-2008
16. Chandra, J.M. and Bahner, M.L. (1988) The effects of inflation and the value of money or some inventory systems. International Journal of Production Economics 23, (4) 723-730.
17. Mandal, B.N. and Phaujdar, S. (1989)An inventory model for Operational Research Society 40, 483-488.
18. Hwang, H. Shinn, S.W. (1997) Retailer’s pricing and lot sizing policy for exponentially deteriorating products under condition of permissible delay in payments. Computers and Operations research 24, 539-547.
19. Jamal, A.M. Sarkar, B.R. and Wang, S. (1997) An ordering policy for deteriorating items with allowable shortage and permissible delay in payment. Journal of Operational Research Society 48, 826-833.
20. Chakrabarty, T., Giri, B.C. and Chaudhuri, K.S. (1998) An EOQ model for items with weibull distribution deterioration, shortages and trended demand and an extension of Philip’s model, Computers and Operations Research 25, (7/8), 649-657.
21. Mondal, M. and Motti, M. (1999) Inventory of damageable items with variable replenishment rate, stock dependent demand and some units in hand. Applied Mathematical Modeling 23, 799- 807.
22. Liao, H.C., Tsai, C.H. and SU, C.T.(2000) An inventory model with deteriorating items under inflation when a delay in payment is permissible International Journal of Production Economics 63, 207-214.
23. Chung, K.J., Huang, Y.F. and Huang, C.K. (2002) The replenishment decision for EOQ inventory model under permissible delay in payments. Opsearch 39, 5 & 6, 327-340.
24. Chung, K.J. and Hwang, Y.F. (2003) The optimal cycle time for EOQ inventory model under permissible delay in payments. International Journal of Production Economics 84, 307-318.
25. Chung, K.J. and Huang, T-S. (2005) The algorithm to the EOQ model for Inventory control and trade credit. Journal of Operational Research Society 42, 16-27.
26. Bharat G., Bansal K.K. [2015] A Deterministic of Some Inventory System For Deteriorating Items With An Inventory Level Dependent Demand Rate. *International Journal of Education and Science Research Review2[6] 94-96*
27. Dye, C.Y. (2002) a deteriorating inventory model with stock dependent demand and partial backlogging under condition of permissible delay in payments. Opsearch 39,(3 & 4).

28. Mishra, V.K. and Singh, L.S. (2010) Deterministic inventory model for deteriorating items with time dependent demand and partial backlogging, Applied Mathematical Sciences Vol.4, no. 72,3611-3619.
29. Mandal, Biswarajan (2013) Inventory model for random deteriorating items with a linear trended in demand and partial backlogging. Research Journal Of Business Management and Accounting. Vol. 2(3), 48-52.
30. Mishra V.K. (2013) Inventory model for instantaneous deteriorating items controlling deterioration rate with time dependent demand and holding cost. Journal of Industrial Engineering and Management, 6(2), 495-506.
31. Chaudhary R & Singh S. R. (2010). " An inventory model with time dependent demand and deterioration under partial backlogging ", International Transactions in Applied Sciences January-March 2011, Volume 3, No. 1, pp. 71-78